Bayesian modelling of an absolute chronology for Egypt's 18th Dynasty by astrophysical and radiocarbon methods

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1. Introduction

From the beginning of modern Egyptology, the establishment of an absolute chronology for Ancient Egypt has been an ambition which has concentrated the minds of many scholars. Pioneers in the field constructed a relative chronology by re-studying Graeco-Roman sources and deciphering ancient Egyptian writing. Firstly, Herodotus's writings were re-translated providing the first pieces of information that led up to Manethon's epitomes, which proposed lists of kings and the lengths of their reigns. These documents led to the first organization of Egyptian time into 31 groups of kings, called dynasties. The second major advance came with the studies of Champollion and his followers, who succeeded in understanding how to read hieroglyphs, the writing of the ancient Egyptians. Thus, Egyptian sources became comprehensible and archaeology brought to light documents of great interest which contained lists of the kings who reigned in Egypt. The Palermo Stone, the Abydos reliefs and the Turin Canon proposed lists of the kings and their order of succession. All these documents, to which are added a host of archaeological discoveries, have permitted Egyptologists to build a relative chronology for ancient Egypt. Henceforth, the 3000 years of dynastic history were divided into six periods. The three "stable" periods, called the Old, Middle and New Kingdoms are separated by three periods when the Egyptian state was divided between the North and the South: the First, Second and Third Intermediate Periods. Following the Manethonian tradition, these periods are organized into dynasties: the Old Kingdom (OK) consists of the Dynasties (Dyns) 3-6, the Middle Kingdom (MK) of the Dyns 11-13 and the New Kingdom (NK) of the Dyns.18-20. Dyns.1-2 precede the Old Kingdom and thus form the so-called Early Dynastic Period (EDP); Dyns.7-10 make up the First Intermediate Period (FIP); Dyns. 13-17, the Second Intermediate Period (SIP); Dyns.21-25, the Third Intermediate Period (TIP). Lastly, the Third Intermediate Period is followed by the Late Period (Dyn.26) and the Graeco-Roman Period. This study will focus on the 18th Dynasty of the NK, for which museum held sufficient material to permit the development of relevant Bayesian models.

In Ancient Egypt, the notion of time was totally different to ours and relied on deep religious principles. Civil time was organized by a calendar based on agricultural seasons, divided into different units of time: a year contained three seasons (Akhet, Peret and Shemu) of four months and each month lasted thirty days. Five feast-days, called epagomenol days, were added at the end of the year. There was no larger unit of time than the year. Historical time was just expressed in reference to the ruling king: as examples, the construction of the Akhmenu of Karnak was described as being built in year 24 of Thutmoses III, and the battle of Qadech took place in year 5 of the reign of Ramses II.

Such an organization of time means that we would need to know the exact succession of all the kings and their precise length of reign, to re-establish an absolute chronology for Ancient Egypt. Unfortunately, our state of historical and archaeological knowledge does not permit the reconstruction of such a long period of history and Egyptologists used to date an event by citing the relevant Kingdom or Intermediate Period and the Dynasty and possibly the king under which the event occurred.

In the present study, we propose a new approach based on a statistical method, called Bayesian modelling, to establish an absolute chronology for Egypt's 18th Dynasty. In this model, which includes different absolute astrophysical and physical methods, radiocarbon dates give the base-information, which is then constrained by results deduced from Sothic and Lunar methods calculated with a Bayesian approach, as well as Egyptian textual sources, and incorporated as *a priori* to the radiocarbon information.

2. Materials and Methods

2.1. Bayesian modelling

A probabilistic approach, called Bayesian statistics, allows us to define observations taking into account our global knowledge on a studied object/event. Bayesian modelling is based on subjective hypotheses that represent the state of the established knowledge when the dating is performed. Such an approach highlights the combination of two systems of time: the first is relative time (represented by the archaeological evidence) and incorporated as *a priori* to a second time, which is absolute time, represented by measurement and called the *likelihood*. The age deduced by combining archaeological and dating information, is the *a posteriori* law, and this distribution has to be accepted or rejected depending on particular criteria that we have to define in advance. Such an approach seems to be particularly appropriate for dating archaeological objects/events since contextual as well as stratigraphic information is usually established before measurement.

In particular, radiocarbon dates are rarely the only information we have on analyzed objects. The archaeological context in which they were found/conserved as well as the correlations between objects (inbuilt age, contemporaneity, position of one object to another, etc…) give extra information. This may improve the precision of the age distributions deduced by radiocarbon dating, by constraining the radiocarbon densities by using archaeological evidence.

So, this Bayesian approach was firstly performed on Lunar dating, by writing our own C++ programme. Then, we modelled radiocarbon measurements using OxCal 4.1, and we used *priors (combine, phase, sequence, gap, interval, boundary, Delta_R),* agreement and convergence factors following the terminology proposed in (Bronk Ramsey, 2009).

2.2. Sothic Dating

Sothic Dating was the first method used by Egyptologists to establish the backbone of an absolute chronology for Ancient Egypt (Borchardt, 1899; Neugebauer, 1929; Parker, 1977; Krauss, 1985). This method is based on the observation of the star Sirius/Sothis, called Sopdet by the ancient Egyptians. Sopdet was an Egyptian goddess who had been known since the first Egyptian dynasty, and who was identified with the star Sothis, the brightest star in the night sky. Each year, Sothis disappears from the sky of an observer in Egypt and reappears about seventy days later. This phenomenon is called a Sothic rising. By pure coincidence, the time of reappearance of this star in the Egyptian sky corresponded with the time of the annual Nile flood. As a result, Sopdet was considered the goddess of fertility. This coincidence led Egyptians to base their calendar on the Sothic rising. When their calendar was created, the first day of the first month of the year was a day close to the Nile flood, and was also the day of the reappearance of Sothis. However, the Egyptian year lasted 365 days whereas the Sothic year was of 365 days plus one quarter. Thus, a gap of one day per four years had been created between this Egyptian civil year and the Sothic year. It meant that four years after the creation of the calendar, the Sothic rising occurred the second day of the year, eight years after, the third day etc.… So, a Sothic cycle is the time which separates two Sothic risings which have occurred the same day in the civil Egyptian calendar. It lasted 1460 Julian years (365*4=1460), which means 1461 Egyptian years. In reality, this value is not totally exact: the Sothic cycle is not fixed because of the own movement of the star and the length of Sothic year evolves in time.

Egyptians were aware of this offset between their civil calendar and the Sothic cycle but never modified it. Thus, they sometimes engraved the correspondence between the heliacal rising of Sothis's star and the date on which it occurred (king, year, month and day). These "Sothic equations" are of the utmost importance because an absolute chronological date for the year of the ruling king can be deduced from them. Sothic documents are attested, in particular a Sothic equation was found on a stone excavated at Elephantine Island, near Aswan. This block refers to an offerings' calendar, and in this document, presently exhibited at the Louvre Museum (E 3910), it is written: *"3th month of the season Shemu, day 28, day of the feast of the coming of Sopdet, …"* which means that a Sothic rising occurred the 28th day of the third month of the season Shemu (Shemu.III.28), under an unknown king. A Roman writer called Censorinus wrote in *De Die Natali* (Censorinus) that the 19 July 138 C.E was the first day of this Egyptian civil year. This association gave us the first piece of information to evaluate absolute dates on which Sothic risings occurred. Then, we used a method, based on the *arcus visionis* estimation. It is the smallest angle difference between the sun and the rising of a star that is necessary for the celestial object to be just visible at its rising by a person on the earth. Our method estimates the absolute date on which the Sothic rising occurred, according to Julian years (Aubourg, 2000).

2.3. Lunar Dating

In parallel with the civil calendar, a lunar calendar was used in ancient Egypt, especially in temples (Parker, 1950 ; Depuydt, 1997). It was based on 12 months of 29 or 30 days, and the first day of the month was the first day of the moon's invisibility. Thus, the length of the month was determined by observation of the moon on the 29th day: if the moon was again visible, the month lasted one more day and the new one begun the day after the following day, but if it was invisible, this following day was considered the first day of the new month.

Some lunar equations are attested in the texts, in particular during the reign of Thutmoses III: they consist in associating one phase of the moon with a date in the Egyptian civil calendar. Now, days and hours of new moons around the reign of Thutmoses III have been tabulated thank to the JPL-HORIZONS software (JPL Horizons) developed by the NASA. With a Bayesian approach, we have determined the more probable dates of observation of new moons attested in Egyptian texts, by synchronizing these lunar equations with the previous Sothic method.

2.4.Radiocarbon Dating

At LMC14 (CEA Saclay, France), two sets of radiocarbon analyzes were carried out on short-life objects conserved at the Louvre museum and archaeologically attributed to a specific reign or short period of the 18th Egyptian Dynasty. Care was taken to sample objects without preservatives. Samples were treated using Saclay's routine pretreatment process for organic material (acid -base-acid procedure consisting of HCl (0.5N, 1 hour, 80°C), NaOH (0.1N, 1 hour, 80°C) and HCl (0.5N, 1 hour, 80°C)). They were then dried and combusted at 900°C with CuO and silver wire. CO_2 produced was collected cryogenically before being reduced to graphite with H_2 and Fe powder at 600°C. Measurements were performed with the AMS method at the ARTEMIS Facility. Calibration and Bayesian modelling were realized using the OxCal4.1 program (Bronk Ramsey, 1995), using IntCal09 curve (Reimer et al, 2009). Radiocarbon results are used to be given with one standard deviation and calibrated ages, with two sigmas deviation. In the modelling, an offset of 19 ± 5 years was added to each radiocarbon date, following the study of the Oxford Laboratory on the reservoir offset in Egypt (Dee et al, 2010) and using the *Delta_R* function (Bronk Ramsey, 2009).

2.4.1.Sennefer's tomb at Deir el-Medineh

Bouquets of flowers found in Sennefer's tomb at Deir el-Medineh by the French Egyptologist Bernard Bruyère (Bruyère, 1929) (Figure 1), were radiocarbon dated. 47 samples were analyzed; we sampled different short-life plants (leaves, twigs, etc..) to ensure the consistency of the dates. They came from seven different bouquets. The archaeological material found inside the tomb shows three burial phases. They occurred between the beginning of the reign of Tutankhamun and the beginning of the reign of Horemheb, which means a period of about 15 years (Bruyère, 1929). In his excavation reports, Bernard Bruyère states that he found all the bouquets at the entrance of the tomb, which means that such material precisely dates the same archaeological event: one of the three phases of burial.

2.4.2.Basketries from Deir el-Medineh

Basketries from the eastern cemetery of Deir el-Medineh (Figure 2) are held at the Louvre Museum. They were found by Bernard Bruyère in the excavations he led between 1928-39, then offered to the Louvre Museum as in recognition of the museum's support for the excavation. The cemetery was used by a relatively modest, almost exclusively female, population. It can be inferred by the study of the archaeological material found inside these tombs (ceramic, scarabs…) that burials in this cemetery occurred at the beginning of the 18th Dynasty. In particular, scarabs with the names of queen Ahmes Nefertari, queen Hatshepsut, and king Thutmoses III are attested (Bruyère, 1937; Pierrat-Bonnefois, 2003). 19 basketries were analyzed by 78 measurements performed on short-life samples of palm, alfa, grapes and seeds, 53 by LMC14 laboratory and 25 by ORAU laboratory. Some of these results have already been incorporated in the 'Radiocarbon-based chronology for dynastic Egypt' proposed by the Oxford laboratory (Bronk Ramsey et al, 2010).

3. Results

3.1. Sothic Dating

The Elephantine Calendar refers a Sothic rising on Shemu III.28 at Elephantine Island, which corresponded to the 328th Egyptian civil day. To elaborate, Figure.3 shows that around 1450 B.C.E, the Sothic rising occurred at Elephantine at about the 10.5 ± 0.75/0.5 Julian July. This 10.5 ± 0.75/0.5 Julian July has to be the 328th or the 327th day of the civil Egyptian calendar, since we have to consider an uncertainty of one day between the observation day and the effective day of the Sothic rising. From Censorinus text, we know that 19 Julian July 138 EC corresponded to Akhet.I.1, from what we calculated that the 11/10 Julian June 138 EC corresponded to Shemu.III.28/27. By subtraction, we deduced that Shemu.III.28/27 corresponded to the 10.5 ± 0.75/0.5 Julian July between 1439 and 1448 BCE, which enable to conclude that this heliacal rising of Sothis took place in 1443.5 ± 4.5 B.C.E (Figure 3). Unfortunately, the year of the ruling king on which this observation was made isn't mentioned, but it may firstly be shown that this block was engraved during the reign of Thutmoses III (Bommas, 2000), which lasted 53 years (Hornung, 2006). Besides, the study of the archaeological context on which the Elephantine calendar was found leads us to the conclusion that the wall on which it was located could not have been engraved before the year 33 of this king. Thus, we deduced that 1443.5 ± 4.5 B.C.E must be between the year 33 and 54 of Thutmoses III (Quiles, 2010).

In addition, for Egypt's 18th Dynasty, successions of kings and lengths of their reigns are relatively well-known, as summarized in TAB. 3. Combining the derived equation with the known lengths of reigns, termini were calculated for each reign of this dynasty. Thus, we deduced that 1568.5 ± 4.5 B.C.E. and 1285.5 ± 4.5 B.C.E. were respectively termini *post* and *ante quos* for the beginning and the end of the 18th Dynasty. These termini were obtained without preferring one Egyptologist's estimations to another.

3.2. Lunar Dating

Thutmoses III relates two lunar equations in his Annals. The first one is attested as the day of the beginning of the Megiddo Battle. He explains: "*Year 23, Ist month of Shemu, day 21, day of the battle of Megiddo, day of the feast of the new moon*" (*Urk*.IV.657.1). The following year, the king inaugurated the Akhmenu, a festival hall he built in the Karnak complex. He said: "*My majesty orders to prepare the ceremony of foundation to come the day of the feast of the new moon, for the act of foundation of this monument, in the year 24, 2nd month of the season Peret, last day*" (*Urk.*IV, 836.2). Knowing the accession date of Thutmoses III in the Egyptian calendar (Shemu.I.4, (*Urk.*IV.180, 15-16)) and being aware that during the New Kingdom, the first year of a king always lasted 365 days, we deduced that these two dates are separated by 649 days. Besides, thanks to Sothic dating we established years 23 and 24 of Thutmoses III were between 1480 and 1448 B.C.E. And by calculating the correspondent day of these two celebrations in the Julian calendar, we deduced that these new moons occurred between 1st and 20th May for year 23, and between 8 and 27 February for year 24 (Quiles, 2010). With JPl-Horizons software, the new moons of February and May between 1480 and 1448 B.C.E. were tabulated. Texts do not however specify whether the Egyptians observed the moon in the morning or in the evening. This information is of prime interest because the day of observation of a new moon could have been different from the actual day (for instance, if Egyptians used to observe the new moon in the morning, a new moon which was sighted at about 9 pm was observed on day D+1 rather than day D.).

For this reason, we have written two models of calculation in C++ language, using a Bayesian approach, to determine the more probable day of observation. As *likelihood*, we have taken a Gaussian law of distribution, in view of the probability of being able to observe the new moon once you know it has occurred. Two *a priori* laws were tested, according to an observation in the morning, or in the evening. These *a priori laws* were defined as Heaviside functions except around the boundaries where the probability of observation was fixed with a Gaussian law (sunrise and sunset). The difficulty was to set boundaries of the hours up to which observation was possible. They were set in reference to the hours of sunrise and sunset (consequently, the boundaries are not the same for the new moon's observation in May or in February). As a result, the *a posteriori* law answered the question: "*knowing the effective hour of the new moon, and postulating a type of observation (morning or evening), what is the probability the observation was done on day D or day D+1?*"

After having tabulated the more probable days for observation of the two new moons (February and May) between 1480 and 1448 B.C.E., we checked the ones separated by 649 days. The sum of results of the two models enables us to propose a set of 12 years that are possible as year 1 for the reign of Thutmoses III. These are summarized in Figure 4. (Quiles, 2010)

3.3. Radiocarbon dating

3.3.1.Sennefer's tomb

In the 47 analyses carried out on bouquets from Sennefer's tomb (TAB.1), five outliers were identified. At first we considered samples coming from the same bouquet as dating the same archaeological object, so having the same age. These ages were combined using *R_Combine* function in OxCal 4.1 (Bronk Ramsey, 1995) so as to get one age per bouquet, and they were then constrained by termini *post* and *ante quos* for, respectively, the accession date of Tutankamun and Horemheb, deduced from Sothic dating (1356.5 ± 4.5 B.C.E. and 1312.5 ± 4.5 B.C.E.). The seven ages obtained are contemporaries and spread out over a period from 1420 to 1260 calBC (Figure 5).

Because the seven bouquets were found one on top of the other at the entrance of the tomb, we can proceed to consider that all the bouquets precisely date to the same archaeological event. The combination of the seven probabilities, constrained by the same termini, leads to the proposition that the bouquets were put out in Sennefer's tomb between 1358 and 1312 calBC (Figure 5).

3.3.2.Basketries from Deir el-Medineh

The 19 basketries from the eastern cemetery of Deir el-Medineh were analyzed by one to nine measures (TAB.2) and 10 outliers were identified. The validated dates have been combined to get one probability density for the age of each object. 14 are in the reign of Thutmoses III whereas four basketries (E 14487, E 14479, E 14617 and E 16397) could be older (around the beginning of the New Kingdom). For specifying the period of activity of this cemetery, we have tried to identify the original tomb of some of the basketries. The study done by Y. Gourlay and the description of the tomb detailed by B. Bruyère permit the finding of the original tomb of nine objects, but it was not possible to proceed for the last ten (Gourlay, 1981 ; Bruyère, 1937). As an example, we observe that four basketries come from tomb 1382, in which three sarcophagi were found (Bruyère, 1937). These three burials did not occur at the same time, so we cannot say that the four basketries have the same age. Moreover, we cannot consider that the basketries precisely date the tombs or a burial phase, because a person could have been buried with a basket that was 10/20 years older. So an archaeological uncertainty of one to two generations has to be applied between the age of the basketries and the ages of burial phases in these tombs.

4. Egypt's 18th Dynasty model

The comparison of dates deduced from Sennefer's tomb and the ones from basketries, synchronized with Sothic and Lunar methods, allow an absolute chronology for Egypt's 18th Dynasty to be modelled with a Bayesian approach.

4.1. Accession date for Tutankhamun

The association of the ages of the Sennefer's bouquets with a historical period is more precise than for the basketries, as the former are associated with the interval from the beginning of the reign of Tutankhamun until the beginning of the reign of Horemheb, which is a period of 15 years, whereas an uncertainty of one to two generations surround basketries. That is why we chose to favor Sennefer's probability density for the construction of our chronology. However, to establish an absolute chronology we need a starting point associated with a precise historical event, like the accession date for Tutankhamun. So, we multiplied the combined density we got for "*one phase of burial in Sennefer's tomb*" by a rectangular function from 1 to 15 years, to simulate a distribution of age for the beginning of the reign of Tutankhamun. This age was then constrained by the termini *post* and *ante quos* for the beginning of this reign, deduced from the Sothic method. We obtained the result that Tutankhamun became king of Egypt between 1359 and 1319 calBC (2σ). This age takes into account archaeological as well as physical uncertainties (Figure.6).

4.2. Accession date for Thutmoses III

The information from Sennefer's tomb is more precise than the attribution of the basketries to Thutmoses III. The length between the beginning of the reign of the latter and Tutankhamun is relatively well known (between 140 to 149 years, TAB.3). That is why, in the same way, we simulated an age for the beginning of the reign of Thutmoses III, from the age of Tutankhamun (1499-1463 calBC (2σ), Figure.6). Then, the ages obtained with the basketries were incorporated in a "phase". Additionally, the years calculated by Lunar dating as possible estimates for year 1 of the reign of Thutmoses III were added in another "phase". We defined these lunar dates by Normal distributed errors of one year. These two phases and the simulated date were integrated in another bigger phase, so as to constraint the simulated age by the two first phases. Finally, the density we get for the accession date of Thutmoses III was constrained by termini *post* and *ante quos* for the beginning of this reign, deduced by the Sothic method. The youngest lunar date was rejected by calculation (agreement factor was too low) and with a set of 11 lunar dates, the model is accepted with an agreement factor of 81. It suggests we consider that the reign of Thutmoses III began between 1499 and 1471 calBC (2σ).

The reign of Thuthmose III has been the central focus of several chronological analyses on the New Kingdom, using astronomical as well as historical approaches (Krauss 1985; Leitz, 1989; Grimal, 1988; Beckerath, 1994; Shaw, 2000; Hornung *et al.*, 2006). The aim of this paper is to provide a new account, using only the most reliable archaeological information in combination with the latest chronometric techniques.

4.3. Accession Dates for each king of Egypt's 18th Dynasty

Consequently, we have obtained two dates for two different reigns of the 18th Dynasty and we have *termini post* and *ante quos* for the beginning and the end of this period, thanks to Sothic dating. That is, we have four corner points. For this period, the succession of the kings and the length of their reigns are well attested in history and archaeology. The majority of them are known with an uncertainty of about one to two years, but three are not as well-established and their uncertainty is of about 10 years. For the first group, we chose to define a probability distribution for the length of the reign by a rectangular function. This affords equal probability to each year of the function. Even if this kind of distribution law is restrictive for the calculation of the model, it represents our state of knowledge. On the other hand, the length of Thutmoses Ist, Thutmoses II and Horemheb's reigns are not as well established. For instance, some Egyptologists affirm that Horemheb's reign lasted 13 years whereas others claim it reached 27 years. That is why we chose to define this probability density using the *Before* and *After* OxCal functions (Bronk Ramsey, 2009). Because the reign lasted at last 13 years, we used the restrictive *After*(13) function. Then, because it could have lasted more that 13, and until 27 years, we combined the former with a *Before*(27+T(5)) function. Such a probability density makes the distribution more flexible. Lengths of Thutmoses Ist and Thutmoses II's reigns were defined using the same functions.

As a result, we have simulated an interval of age per king by the multiplication of these relationships on the length of the reigns, to one of the two simulated radiocarbon's densities we got for Thutmoses III and Tutankhamun (Figure 7 and TAB. 3).

4.4. Implications on our model

Our model calculated an age distribution for the accession date of each king of Egypt's 18th Dynasty, which synchronizes radiocarbon, Sothic, lunar, historical and archaeological information. Intervals are about 32 (Thutmoses III) to 45 (Horemheb) years with 95% probability (TAB. 3). They are shorter for the beginning of the dynasty which means the terminus *post quem* we calculated is relatively precise. For the end of the dynasty, the model is more flexible because intervals are larger. This can be explained by the fact the terminus *ante quem* we use is lower. It was calculated thanks to Sothic method, combined with historical information on the lengths of reigns. It means we have summed the maximum lengths for each reign after Thutmoses III, and taken as a starting point the first possible year possible for estimating the year of observation of the Sothic rising engraved on the Elephantine Calendar. Anyway, given our state of knowledge, we could not calculate a higher terminus *ante quem* without preferring one Egyptologist's interpretation over another. But, that could show that a couple of reigns were shorter that what we used, in particular for reigns for which the length is not well established. Another explanation would be the Sothic rising's observation could have occurred closer to the end of Thutmoses III's reign, rather than around year 33. But, this can not be definitively established.

This model may be further improved if we get information on another constraint (like a more precise knowledge on the length of a reign, another astrophysical observation, etc…). But, at present, modelled intervals are strongly correlated, which means new information on just one reign will significantly modify each interval. Getting more dates for different reigns that Thutmoses III and Tutankhamun seems to be the best solution to get an "asymptotic model". The more constraints we establish, the less intervals will vary.

5. Conclusions

Radiocarbon dating is a physical method which can be used to determine probability densities for the age of archaeological events. Thanks to historical and archaeological evidence, the succession of kings and lengths of their reigns for the 18th Egyptian Dynasty are relatively well-established. Besides, we have astrophysical equations attested in Egyptians texts which allow us to calculate anchor points in

Egyptian chronology. The first one is Sothic dating and in this paper we have shown, firstly, that these equations allow us to minimize uncertainties by using *arcus visionis* method. As a result, we achieved a relatively precise anchor point for the reign of Thutmoses III. Combined with our knowledge on the succession of the kings and the lengths of their reigns, we were able to fix termini for each of the reigns of the 18th Dynasty. Lunar dates were then a second source of information and we developed a Bayesian model which calculates the more probable days of observations of new moons, given the type of observation (morning or evening). By incorporating the two lunar equations attested in the Annals of Thutmoses III, we deduced a set of 12 years as possible estimates for the accession date of this king. Then, two radiocarbon studies were carried out at LMC14 and ORAU on samples archaeologically attributed to a period Tutankhamun-Horemheb and Thutmoses III. The first leads to precisely dating one phase of burial in Sennefer's tomb, and from that result we simulate a date for the beginning of Tutankhamun, then for Thutmoses III. The second was constrained by densities we obtained on basketries from Deir el-Medineh, and by lunar dates. According to our knowledge on the length of each reign of the 18th Dynasty, we finally determined a probability distribution for the length of each reign of this period, which allows the simulation of a period for the accession date of each king of this dynasty. These temporal probability densities incorporate Sothic, Lunar, radiocarbon, archaeological and historical information.

Our model was applied on a reliable Egyptian period and results we got are in perfect agreement with dates previously deduced by some Egyptologists from methods unused here. This allows us to conclude that such an approach should certainly be applied to older Egyptian periods, for which the textual sources are less extensive.

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Figure Captions:

Figure 1: one of the seven bouquets from Sennefer's tomb (E 14000), held at the Louvre Museum

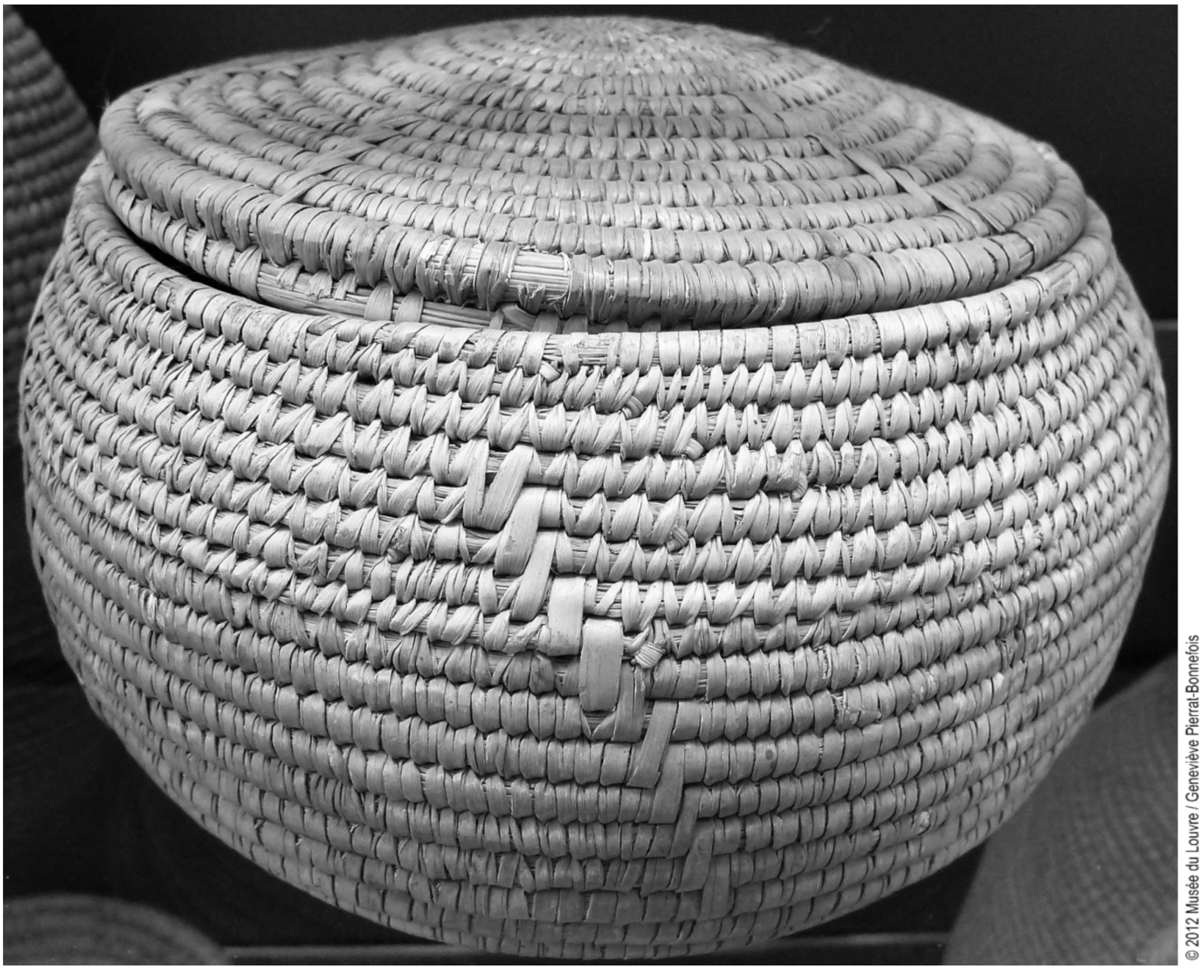

Figure 2: Basketry E 16391, held at the Louvre Museum. Two ^{14}C dates were performed on palm and halfa samples and the deduced combined density shows that this object was fabricated during the reign of Thutmoses III.

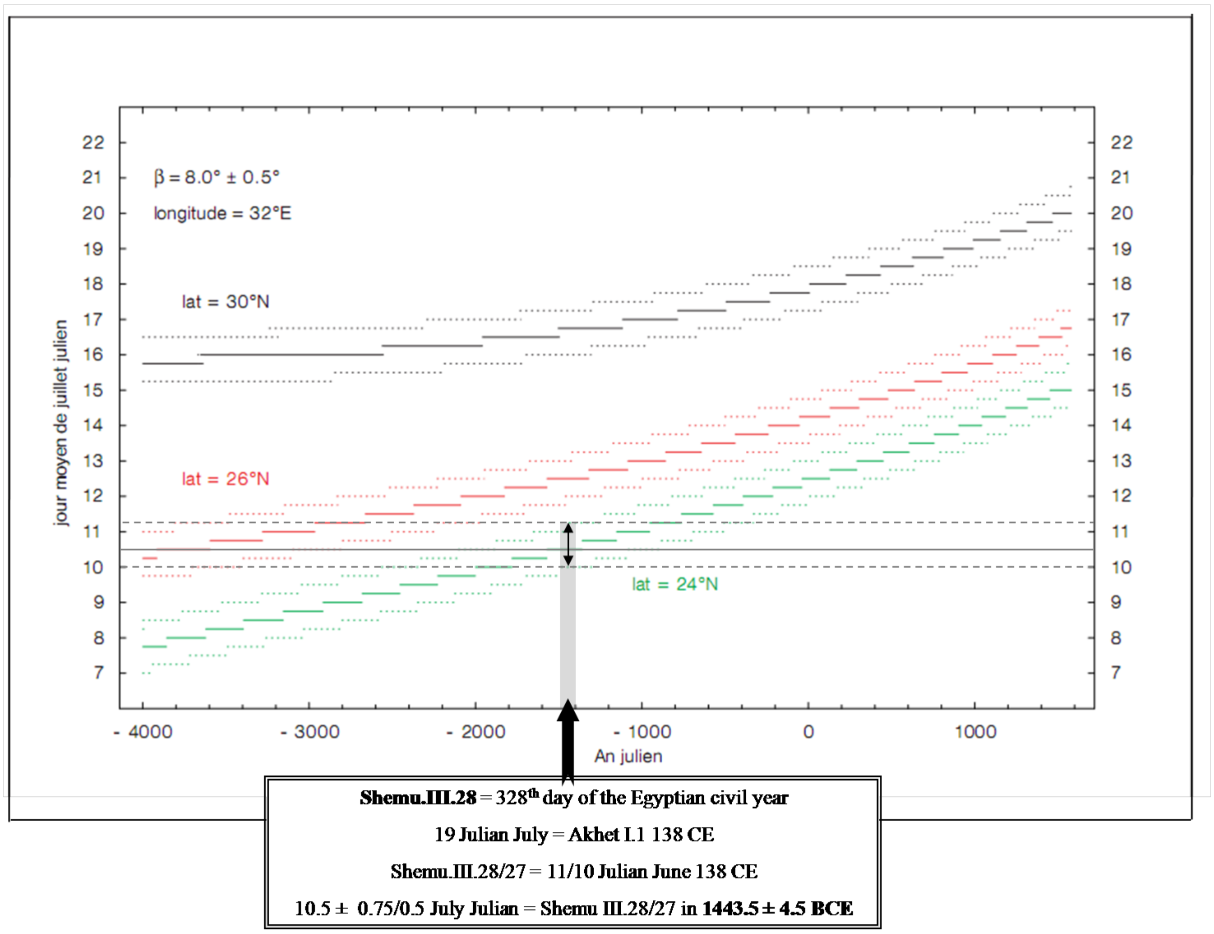

Figure 3: Determination of the Sothic rising's day in Julian July, from (Aubourg, 2000, p. 46). We can observe on this graphic that around 1450 B.C.E, Sothic rising occurred at 24°N, at 10.5 ± 0.75/0.5 Julian July. Because the 19 Julian July 138 EC corresponded to Akhet.I.1, we deduced that 10.5 ± 0.75/0.5 corresponded to Shemu.II. 28/27 from 1448 to 1339 B.C.E.

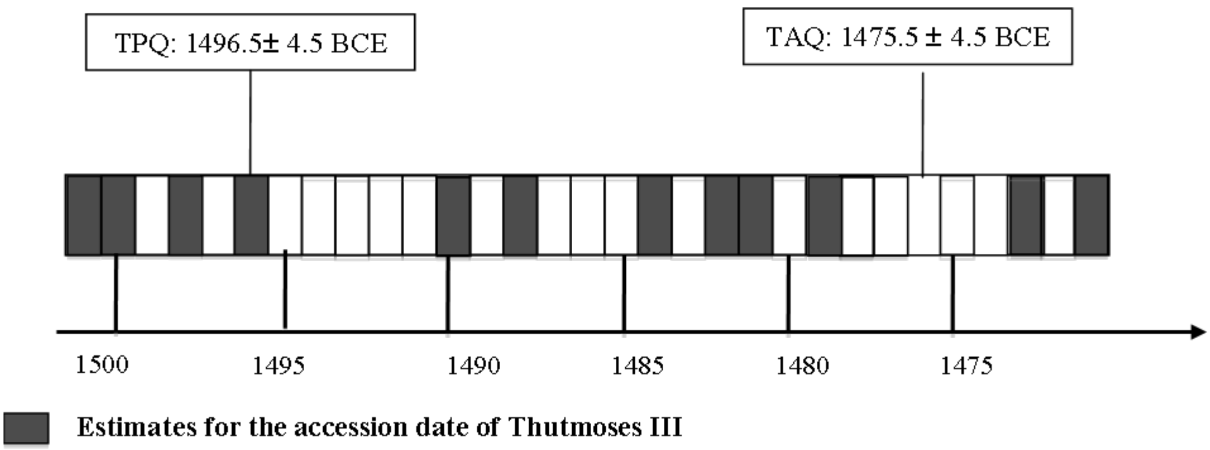

Figure 4: Results of Bayesian modelling of lunar dates attested in *Thutmoses III Annals*. A set of 12 estimates are given for the accession date of this king (in dark).

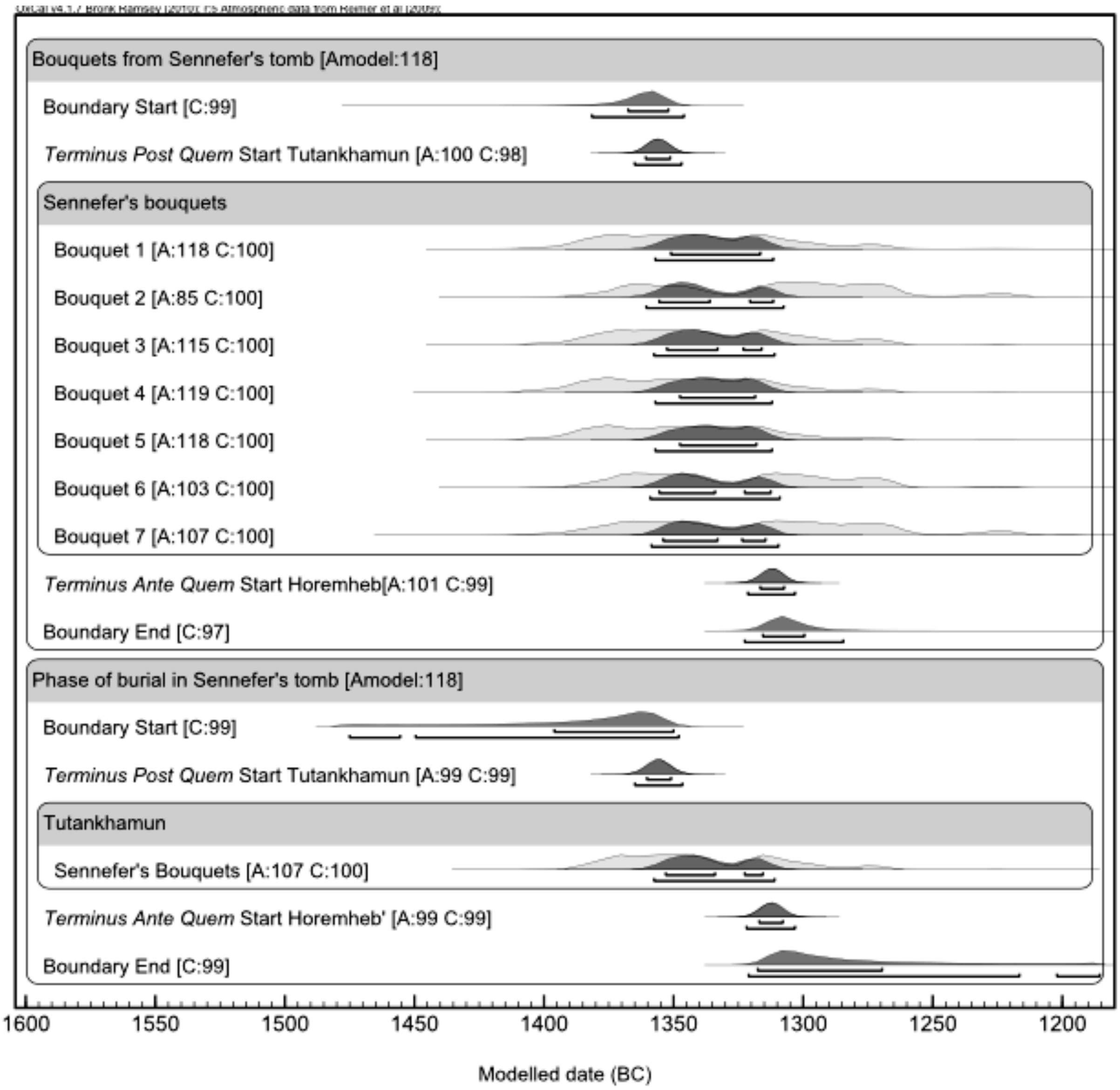

Figure 5: Modelled age for each bouquet from Sennefer's tomb (upper). The seven bouquets combined age stands for the event "*one phase of burial in the Sennefer's tomb*", which occurred between 1358 and 1312 calBC (2σ). *Termini Post and Ante Quos* are deduced from Sothic method. *A posteriori* laws are in dark, likelihood in grey.

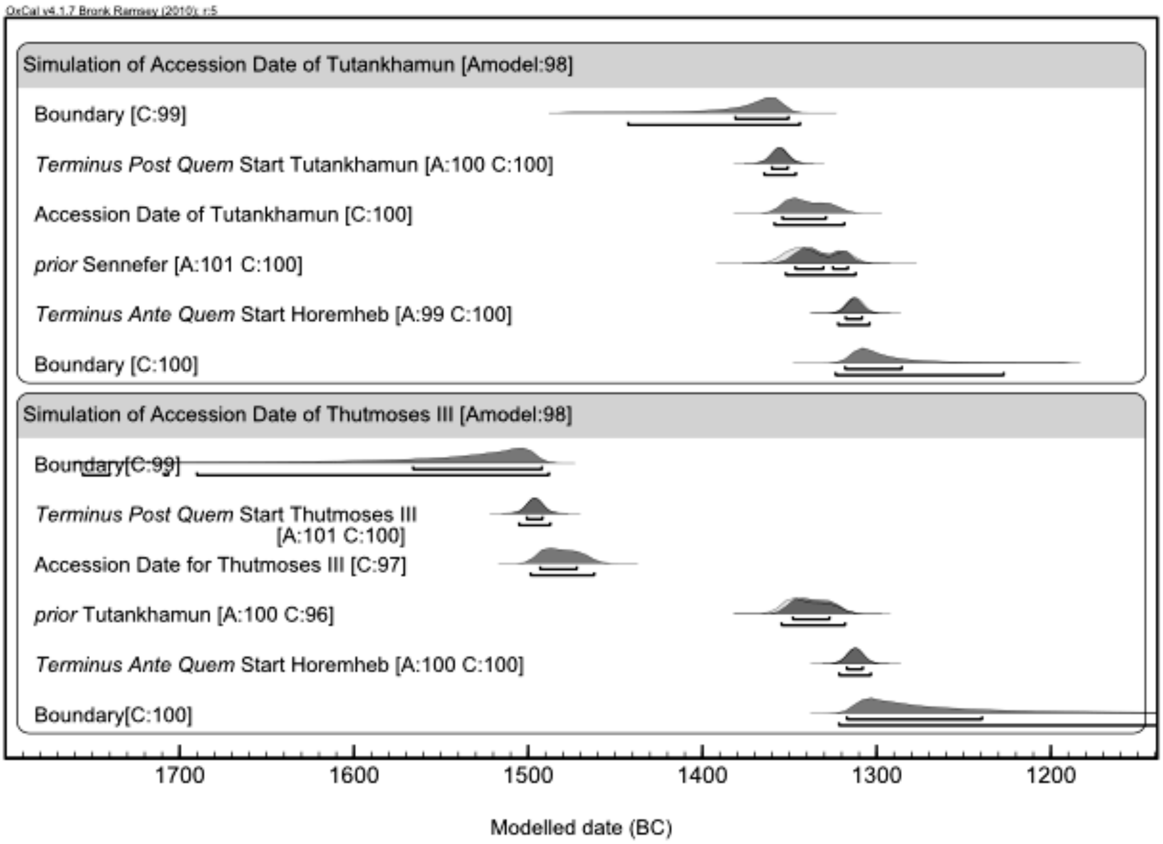

Figure 6: Simulations of Tutankhamun (upper) and Thutmoses III (down) Accession Dates, from the Sennefer density obtained Figure 5. We deduced that Tutankhamun begun king in the period 1359-1319 calBC (2σ), and Thutmoses III between 1499 and 1463 calBC (2σ).

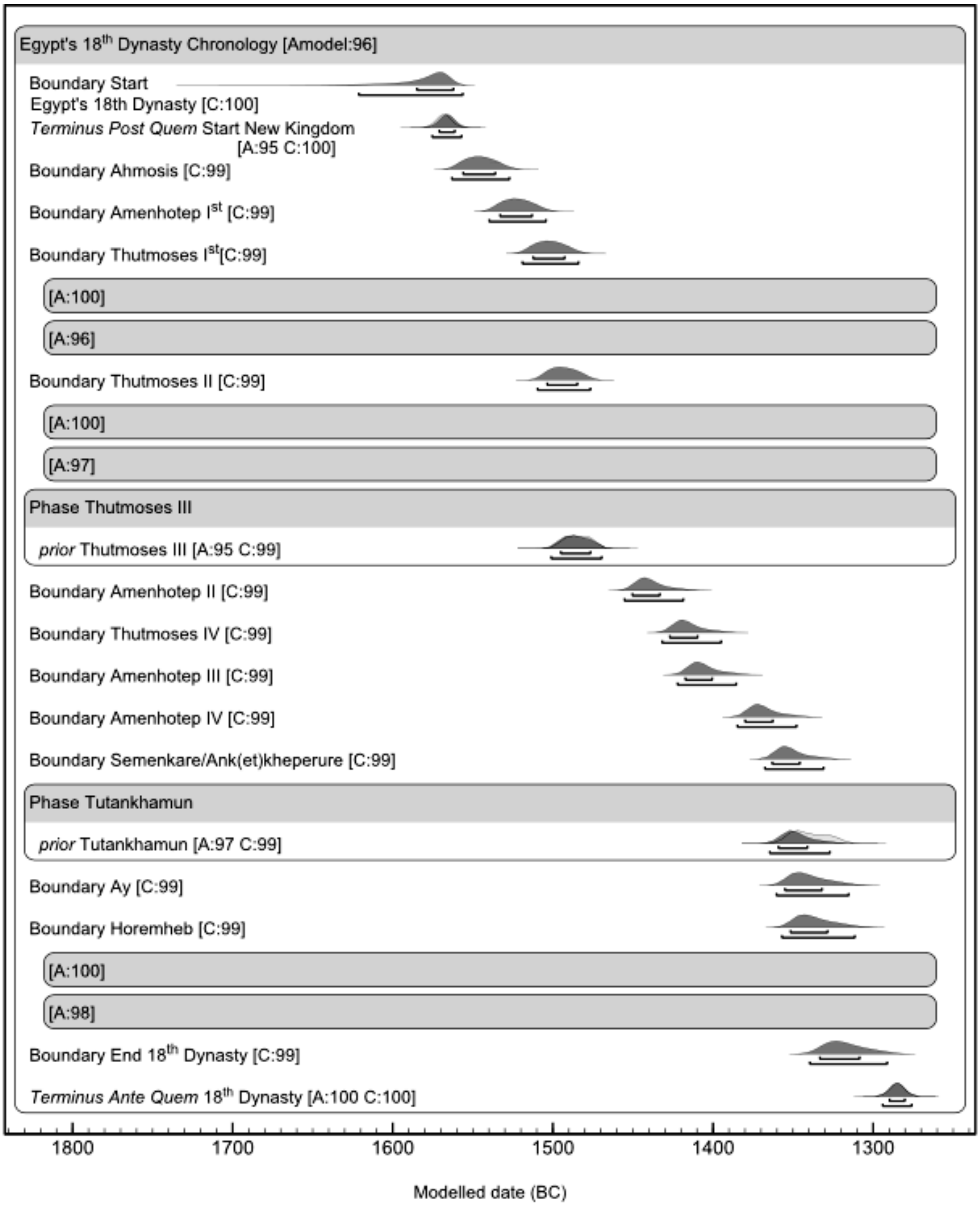

Figure 7: Simulations of kings Accession dates for Egypt's 18th Dynasty. Deduced intervals are about 32 (Thutmoses III) to 45 (Horemheb) with 95% probability.

Sample No.	DAE* No.	Type	mg C	$\delta^{13}C$	Radiocarbon age (BP)		
SacA 15966	E 14000 Bouquet 1	halfa grass	0.70	-28.4	3101	±	34
SacA 18758			0.22	-26.3	3128	±	26
SacA 15967		leaf	1.30	-29.5	3123	±	39
SacA 18759			0.22	-29.1	3089	±	26
SacA 15968		reed	0.48	-25.8	3047	±	36
SacA 18760			0.25	-25.2	3042	±	29
SacA 15969	E 14000 Bouquet 2	leaf	1.20	-30.3	3031	±	36
SacA 18761			0.38	-34.1	2990	±	44
SacA 15970		twig	0.79	-30.4	3024	±	37
SacA 18762			0.22	-32.1	3085	±	26
SacA 15971		halfa grass or reed	0.77	-26.6	3079	±	35
SacA 18763			0.22	-25.5	3170	±	26
SacA 15972		reed	1.20	-30.0	166	±	30
SacA 18764			0.71	-30.6	-94	±	43
SacA 15973	E 14000 Bouquet 3	twig	1.10	-28.6	3029	±	35
SacA 18765			0.22	-27.1	3060	±	26
SacA 15974		leaf	1.40	-30.8	3040	±	36
SacA 18766			0.23	-32.4	2990	±	26
SacA 15975		reed	1.00	-25.4	3088	±	33
SacA 18767			0.21	-22.4	3113	±	25
SacA 15976	E 14000 Bouquet 4	MO	0.91	-28.3	3015	±	34
SacA 18768			0.86	-25.7	3137	±	26
SacA 15977		reed	0.83	-25.2	3044	±	37
SacA 18769			0.79	-24.6	3131	±	28
SacA 15978		reed	0.31	-25.5	2999	±	33
SacA 15979		leaf	0.90	-30.2	3046	±	36
SacA 18770			0.60	-30.5	3120	±	27
SacA 15980	E 14000 Bouquet 5	OM	1.10	-26.9	3100	±	41
SacA 18771			0.86	-25.9	3170	±	25
SacA 15981		reed	0.75	-27.6	3076	±	35
SacA 18772			1.20	-27,0	2924	±	27
SacA 15982		leaf	1.50	-30.3	3135	±	39
SacA 18773			0.50	-33.2	3049	±	26
SacA 15983		twig	0.60	-26.9	1587	±	34
SacA 18774			0.23	-27.6	1178	±	29
SacA 15984		twig	1.22	-27.9	3044	±	36
SacA 18775			0.86	-29.8	3116	±	29
SacA 15985	E 14000 Bouquet 6	OM	1.09	-30.4	3004	±	34
SacA 18776			0.24	-25.9	2984	±	28
SacA 15986		OM	1.19	-28.3	3104	±	34
SacA 15987		wood ?	1.05	-28.4	3055	±	33
SacA 18778			0.24	-27.3	3089	±	28
SacA 15988		leaf	1.35	-29.2	3041	±	34
SacA 18779			0.33	-36.9	2864	±	38
SacA 15989	E 14000 Bouquet 7	twig	1.28	-27.9	3086	±	35
SacA 18780			0.23	-25.3	3190	±	28
SacA 15990		leaf	1.47	-27.7	3036	±	37

TAB 1: Results of 47 samples from Sennefer Tomb undergoing AMS radiocarbon dating. Outliers are identified in grey (*Département des Antiquités égyptiennes, Musée du Louvre).

Sample No.	DAE* No.	Type	mg C	$\delta^{13}C$	Radiocarbon age (BP)		
SacA 11129	E 14477	Palm frond	0.50	-28.6	3164	±	28
SacA 11130		Palm frond	0.60	-26.8	3202	±	23
SacA 11131		Textile	0.70	-22.7	3231	±	22
SacA 11132		Date	0.90	-16.8	3171	±	36
OxA 19448		Palm frond	2.10	-27.1	3245	±	30
OxA 19449		Palm frond	1.80	-26.1	3275	±	31
OxA 19450		Textile	2.14	-23.7	3291	±	31
OxA 19451		Textile	1.83	-23.9	3237	±	30
SacA 11134	E 14489	Palm frond	0.77	-28	3226	±	22
SacA 11135		Textile	1.01	-27.5	3097	±	21
SacA 16397		Halfa grass	0.39	-10.1	3202	±	25
SacA 16398		Palm frond	0.44	-29,6	3232	±	26
SacA 11137	E 16390	Palm frond	0.84	-29.3	3170	±	25
SacA 11138		Halfa grass	0.85	-11.3	3260	±	30
SacA 11139	E 16394	Halfa grass	0.80	-10.8	3224	±	24
SacA 16399		Halfa grass	0.63	-10.7	3188	±	23
OxA 19452		Halfa grass	1.60	-10.7	3227	±	30
SacA 11140	E 16396	Halfa grass	0.95	-13.2	3250	±	25
SacA 11141		Palm frond	1.00	-25.6	170	±	25
SacA 16400		Halfa grass	0.52	-12.5	3191	±	24
SacA 11143	E 16397	Halfa grass	0.80	-26.0	165	±	21
SacA 16401		Halfa grass	0.38	-13.4	3271	±	29
OxA 19146		Halfa grass	1.60	-10.7	127	±	24
SacA 11144	E 16391	Halfa grass	0.08	-10.6	3377	±	53
SacA 11145		Palm frond	0.46	-26.2	3196	±	22
SacA 11148	E 14488	Halfa grass	0.25	-17.6	3089	±	31
SacA 11149		Palm frond	0.80	-19.4	3280	±	26
SacA 16405		Halfa grass	0.86	-10.4	3219	±	25
SacA 16406		Plant	0.42	-15.8	3163	±	28
OxA 19453		Halfa grass	1.70	-12	3264	±	29
OxA 19480		Palm frond	1.90	-24.4	3251	±	26
SacA 11150	E 14491	Halfa grass	0.30	-8.2	3185	±	28
SacA 11151		Halfa grass	0.90	-17.3	260	±	24
SacA 16411		Halfa grass	0.44	-13.4	3193	±	27
OxA X-2287		Halfa grass	0.40	-8.7	3283	±	33
OxA 19149		Halfa grass	0.90	-22.1	165	±	23
SacA 11152	E 14480	Halfa grass	0.60	-10.2	3178	±	25
SacA 11153		Palm frond	0.50	-28.1	3261	±	28
OxA 19481		Halfa grass	1.70	-10.1	3233	±	25
OxA 19150		Palm frond	1.00	-26.3	3153	±	27
SacA 11154	E 14487	Palm frond	1.00	-23.9	3304	±	25
SacA 11155		Halfa grass	0.60	-12.4	3041	±	25
OxA 19482		Palm frond	1.50	-25.3	3277	±	26
SacA 11156	E 14479	Halfa grass	0.70	-11.8	3198	±	25
SacA 11158		Textile	0.80	-25.2	3284	±	22
SacA 11159		Palm frond	0.80	-24.8	3305	±	23
SacA 16409		Halfa grass	0.69	-10.3	3259	±	24
SacA 16410		Halfa grass	0.59	-10.5	3214	±	24
OxA 19151		Halfa grass	1.20	-10.3	3107	±	27
OxA 19581		Halfa grass	2.10	-9.9	3258	±	26
OxA 19483		Textile	1.70	-24.6	3226	±	26
OxA 19484		Palm frond	2,00	-25.2	3257	±	26
SacA 11160		Halfa grass	0.70	-10.7	3200	±	22

SacA 11161	E 14483	Palm frond	0.50	-29	3207	±	31
OxA 19485		Halfa grass	1.90	-10.6	3262	±	25
OxA 19152		Palm frond	0.80	-24.4	3249	±	28
OxA 19486		Cyperus Papyrus	2.10	-24.3	182	±	22
SacA 11162	E 16401	Cyperus Papyrus	0.60	-26.8	178	±	21
SacA 11163		Palm frond	0.70	-25.1	3283	±	22
SacA 11164		Halfa grass	0.60	-10.5	3161	±	22
SacA 16407		Halfa grass	0.40	-13.3	3234	±	31
SacA 11165	E 16393	Halfa grass	0.18	-15.4	3197	±	28
SacA 11166		Palm frond	0.55	-25.2	3155	±	25
SacA 11167	E 14617	Grape	0.72	-27.8	3161	±	25
SacA 11168		Plant remains	0.20	9.5	3756	±	61
SacA 11169		Halfa grass	1.00	-21.8	112	±	18
SacA 11170		Textile	1.00	-24.2	3285	±	22
OxA X-2287		Plant remains	0.40	-22.4	3333	±	33
OxA 19153		Halfa grass	1.10	-22.1	110	±	23
OxA 19154		Textile	1.10	-24.5	3209	±	28
SacA 11171	E 16425	Grape	1.30	-27.8	3241	±	26
SacA 11172		Textile	0.42	-25.8	3192	±	23
SacA 16402	E 14665	Palm frond	0.72	-25.8	3234	±	25
SacA 16403		Palm frond	0.53	-28.7	3200	±	23
SacA 16404		Palm frond	0.50	-28.3	3232	±	25
OxA 19147		Halfa grass	0.50	-10.6	3261	±	32
OxA 19148		Palm frond	0.90	-23.4	3186	±	28
SacA 16408	E 14487	Halfa grass	0.45	-12.9	3243	±	25

TAB 2: Results of 78 basketries samples from the eastern cemetery of Deir el-Medineh, undergoing AMS radiocarbon dating at LMC14 (SacA-53) and ORAU (OxA and OxA X (small carbon mass sample)-25) laboratories. (*Département des Antiquités égyptiennes, Musée du Louvre). Outliers are identified in grey.

King of 18th Dynasty	Length of reigns *Principally from (Hornung et al, 2006)*	Distribution Law on length of reign	Modelled Interval for accession date		
			68 % (calBC)	95% (calBC)	Convergence
Ahmosis	21-25	U(21,25)	1557-1537	1564 – 1528	99
Amenhotep Ist	20-21	U(20,21)	1533-1514	1540 – 1505	99
Thutmoses Ist	3-13	*After*(3)&*Before*(13+T(5))	1513-1493	1520 – 1485	99
Thutmoses II	2-13	*After*(2)&*Before*(13+T(5))	1504-1485	1510 – 1477	99
Thutmoses III	53	*Prior* (Simulated age/Lunar Dates/ basketries)	1496-1477	1502 – 1470	99
Amenhotep II	22-25	U(22,25)	1451-1434	1456 – 1419	99
Thutmoses IV	9-10	U(9,10)	1427-1410	1432 – 1395	99
Amenhotep III	37-38	U(37,38)	1418-1401	1423 – 1386	99
Amenhotep IV	16-18	U(16,18)	1380-1363	1385 – 1348	99
Semenkhare/ Ank(et)kheperure	3-5	U(3,5)	1363-1346	1368 – 1331	99
Tutankhamun	9-10	*Prior*	1360-1342	1365 – 1328	99
Ay	3-4	U(3,4)	1356-1332	1361 – 1316	99
Horemheb	13-27	*After*(13)&*Before*(27+T(5))	1352-1329	1357 – 1312	99
End of 18th dynasty	-	-	1334-1309	1340 – 1292	99

TAB. 3: Establishment of an absolute chronology for kings' accession dates of Egypt's 18th dynasty (1σ and 2 σ).